# Temporal regularities of changing magnetic field generation modes in the model of the $\alpha\Omega$-dynamo.

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Paratunka, 2020

## Goal:

Using the system in the framework of the accepted low-mode model $\alpha\Omega$ - dynamo with an additive correction for the intensity of the $\alpha$ - effect, the question of changing the modes of magnetic field generation depending on the time characteristics of the function core is investigated.

## Problems:

1. Determine the time characteristics of the function J(t) depending on the values of the exponent n and the scale factor b.
2. Investigate the modes of magnetic field generation.
3. Conduct a comparative analysis of the obtained phase portraits.

In the Boussinesq approximation, the MHD system takes the following form

$$
\begin{aligned}
&\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} + \mathbf{f}_c = \nu\Delta\mathbf{v} - \frac{1}{\rho_0}\nabla P - \mathbf{f}_K + \mathbf{f}_{out} + \mathbf{f}_L, \\
&\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \nabla\times(\alpha(r,\theta)\,\mathbf{B}) + \nu_m\Delta\mathbf{B}, \\
&\nabla\mathbf{v} = 0, \\
&\nabla\mathbf{B} = 0, \\
&\mathbf{v}(\mathbf{r}_1) = \mathbf{v}(\mathbf{r}_2) = \mathbf{0},
\end{aligned}
\qquad (1)
$$

where $P$ is pressure,
$\rho_0 = 7\cdot 10^3$ kg/m$^3$ – density,
$\nu$ – kinematic viscosity (limits of change $10^{-6} \div 10^2$ m$^2$/sec),
$\nu_m$ – magnetic viscosity (varies within $1 \div 20$ m$^2$/sec),
$\mathbf{f}_{out}$ – mass density of the field of external forces (source of poloidal velocity),
$\mathbf{r}_1$ and $\mathbf{r}_2$ – radii-vectors of the inner and outer boundaries of the spherical shell of the liquid core.

The mass density of the Coriolis force

$$\mathbf{f}_K = 2\mathbf{\Omega} \times \mathbf{v}, \tag{2}$$

The mass density of the Lorentz forces

$$\mathbf{f}_L = \frac{1}{\rho_0 \mu_0 \mu} (\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{3}$$

The centrifugal acceleration of forces

$$\mathbf{f}_c = \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}), \tag{4}$$

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m is magnetic constant, $\mu = 1$ – magnetic permeability, the angular velocity $\Omega_0$ varies within $10^{-1} \div 10$ 1/sec.

The change in the intensity of $\alpha_0$ over time under the action of external forces is determined by entering the dynamic system (1) additive correction as a function Z(t)

$$\begin{aligned}
&\frac{\partial \mathbf{v}}{\partial t} = P_m \Delta \mathbf{v} - \nabla P - E^{-1} P_m (\mathbf{e}_z \times \mathbf{v}) + \mathbf{f}_{out} + (\nabla \times \mathbf{B}) \times \mathbf{B}, \\
&\frac{\partial \mathbf{B}}{\partial t} = Re_m [\nabla \times (\mathbf{v} \times \mathbf{B})] + (R_\alpha - Z(t))[\nabla \times \alpha'(r, \theta) \mathbf{B})] + \Delta \mathbf{B}, \\
&\nabla \mathbf{v} = 0, \\
&\nabla \mathbf{B} = 0, \\
&\mathbf{v}\left(\frac{\mathbf{r}_1}{r_2}\right) = \mathbf{v}(\mathbf{e}_2) = \mathbf{0},
\end{aligned} \tag{5}$$

where

$$Z(t) = \int_0^t J(t - \tau) Q\big(\mathbf{B}(\tau), \mathbf{B}(\tau)\big) d\tau. \tag{6}$$

The low-mode approximation that includes the minimum number of modes sufficient to obtain reversals in the model $\alpha\Omega$-dynamo with variable intensity $\alpha$-generator is used for the numerical implementation of the system (5)

$$\mathbf{v} = u(t)\mathbf{v_0}(\mathbf{r}) = u(t)(\alpha_1\mathbf{v}^T_{0,1,0} + \alpha_2\mathbf{v}^P_{0,2,0} + \alpha_3\mathbf{v}^T_{0,3,0} + \alpha_{11}\mathbf{v}^T_{1,1,0} + \alpha_{13}\mathbf{v}^T_{1,3,0}), \tag{7}$$

$$\mathbf{B} = B_2^T(t)\mathbf{B}^T_{0,2,0}(\mathbf{r}) + B_1^P(t)\mathbf{B}^P_{0,1,0}(\mathbf{r}) + B_3^P(t)\mathbf{B}^P_{0,3,0}(\mathbf{r}), \tag{8}$$

where $\mathbf{v}_0(\mathbf{r})$ is Poincare mode represented as an expansion in a hilberoth subspace such that $|\mathbf{v}_0(\mathbf{r})| = 1$, $u(t)$ – the velocity amplitude,
$\mathbf{B}^P_{0,1,0}(\mathbf{r})$ – the dipole component of the magnetic field, which under the influence of differential rotation generates a toroidal $\mathbf{B}^T_{0,2,0}(\mathbf{r})$ and poloidal $\mathbf{B}^P_{0,3,0}(\mathbf{r})$ components[3]. The components of the velocity field and magnetic field are considered independent.

The function $Z(t)$ (6) is set as

$$Z(t) = \int_0^t \underbrace{(t-\tau)^n e^{-b(t-\tau)}}_{kernel\ \ J(t-\tau)} \mathbf{B}^2(\tau) d\tau, \quad (9)$$

the maximum of the kernel $J(t) = t^n e^{-bt}$ is shifted by the value $t_0$ (Fig. 1), i.e., the effect of the process that suppresses the intensity of the $\alpha$-effect is turned on at the time shifted by the final delay time.

The final waiting time $t_m = t_1 - t_0$ is determined from the ratio

$$\int_{t_0}^{t_1} t^n e^{-bt} dt = 0.95 \int_{t_0}^{\infty} t^n e^{-bt} dt, \quad (10)$$

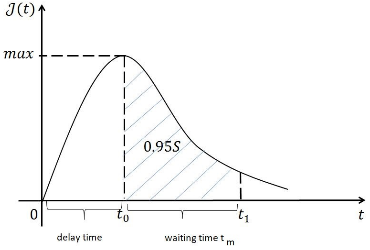


Figure 1: The exponential-power function $J(t)$ graph.

geometrically, this means that 95% of the area under the graph of the function $J(t)$ on the segment $[t_0, \ \infty)$ is concentrated on the segment $[t_0, \ t_1]$.

After applying the Galerkin method, we get the system

$$\begin{aligned}
\frac{\partial u}{\partial t} &= -P_m u(t) \sum_k \alpha_k^2 \lambda_k + f_{out} + \sum_{i,\,j,\,k} \alpha_i L_{ijk} B_j B_k, \\
\frac{\partial B_i}{\partial t} &= Re_m u(t) \sum_{j,\,k} \alpha_j W_{ijk} B_k - \mu_i B_i + (R_\alpha - Z_n) \sum_k W_{ik}^{\alpha} B_k, \\
\frac{\partial Z_0}{\partial t} &= \sum_k B_k^2 - bZ_0, \\
\frac{\partial Z_n}{\partial t} &= nZ_{n-1} - bZ_n, \ \ n = 1,\, 2.
\end{aligned} \tag{11}$$

where $\mu_i$ is the coefficient of viscous dissipation, $\lambda_i$ is the eigenvalues of the Poincare mode, and the coefficients $L_{ijk}, W_{ijk}, W_{ij}^{\alpha}$ are the volume integrals of the fields under consideration.

$$u(0)=1,\ B_2^T(0)=0,\ B_1^P(0)=1,\ B_3^P(0)=0,\ Z_0(0)=0,\ Z_n(0)=0, \qquad (12)$$

The control parameters of the model: the magnetic Reynolds number $Re_m$ was set in the range $(0, 1000]$ and the amplitude of $\alpha$-effect $R_\alpha$ was set in the range $(0,\ 100]$.
The calculations were carried out for the values of the scale factor
$b = 0.1,\ 0.5,\ 1,\ 5,\ 10$ and the values of the exponent $n = 1,\ 2$.
Both methods for given parameters of the model gave identical results, which are presented on the phase plane of the parameters $Re_m,\ R_\alpha$ (Fig. 2 b, Fig. 3, Fig. 4).

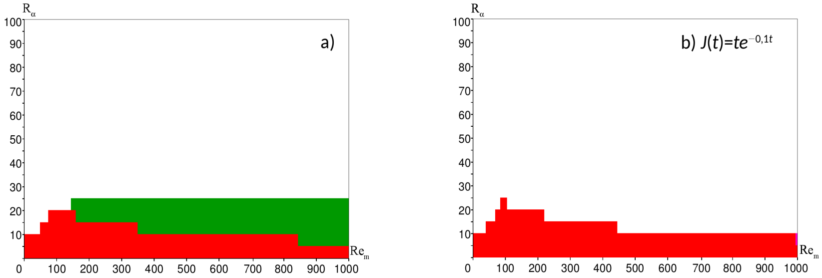


Figure 2: The nature of magnetic field generation depending on the parameters $R_\alpha$ and $Re_m$. The intensity of the $\alpha$-effect: a) constant – $\alpha_0$; in other cases is determined by the function $Z(t)$ with the kernel $J(t)$. The white region is the generation of the increases infinitely magnetic field, the red one – is the generation of a damped magnetic field, the green is the steady mode of magnetic field generation, the blue is the steady-state mode, the yellow is the vacillation, the lilac is the dynamo-burst.

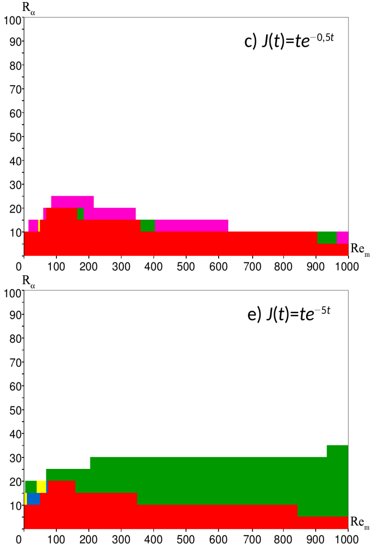


Figure 3: The nature of magnetic field generation depending on the parameters $R_\alpha$ and $Re_m$. The intensity of the $\alpha$-effect is determined by the function $Z(t)$ with the kernel $J(t)$.

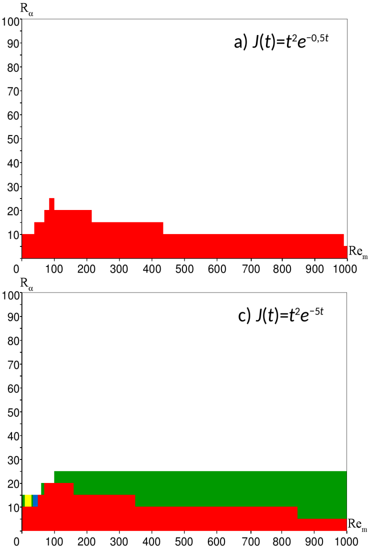


Figure 4: The nature of magnetic field generation depending on the parameters $R_\alpha$ and $Re_m$. The intensity $\alpha$-effect defined by the function $Z(t)$ with the kernel $J(t)$.

Table 1: Time characteristics of the function $J(t)$ depending on the values of the exponent $n$ and the scale coefficient $b$

| $n$ | $b$ | delay time $t_0$ | waiting time $t_m$ |
|---|---|---|---|
| 1 | 0.1 | 10 | 37.695207 |
| | 0.5 | 2 | 7.539041 |
| | 1 | 1 | 3.769521 |
| | 5 | 0.2 | 0.753904 |
| | 10 | 0.1 | 0.376952 |
| 2 | 0.1 | 20 | 44.675701 |
| | 0.5 | 4 | 8.935140 |
| | 1 | 2 | 4.467570 |
| | 5 | 0.4 | 0.893514 |
| | 10 | 0.2 | 0.446757 |

Therefore, when the same in the rest of the conditions of the numerical experiment the variety of modes generating the magnetic field in a slightly varying velocity field increases with scale factor $b$, when decrease while gradually increasing impact of the process $Z(t)$ and the force of the impact.

## Conclusions

- In the framework of the accepted small-mode model $\alpha\Omega$-dynamo, a dimensionless MHD system with an additive correction for the intensity of the $\alpha$-effect is considered under the assumption of axial symmetry of the velocity field and the magnetic field.
- The effect of the turbulent $\alpha$-effect is determined by a function $Z(t)$ with an exponential power core of the form $t^n e^{-bt}$, which specifies a process with a delay time of $t_0$ and a limited waiting time of $t_m$.
- The introduction of the process $Z(t)$ into the MHD system leads to the appearance of new modes of magnetic field generation, including inversions, in comparison with the case of constant intensity $\alpha_0$.
- An increase in the exponent of $n$ is associated with an increase in the delay time $t_0$ and leads to an expansion of the area of unlimited oscillations with an increase in the scale coefficient $b$.
- An increase in the values of the time characteristics of the exponential power kernel $J(t)$ reduces the number of generation modes with a predominance of an unlimited increase in the magnetic field, but practically does not affect the area of damped oscillations, only slightly increases it at $b < 1$.

# References


- M. Steenbek, F. Krause, Astron. Nachr. **291**, 49-84 (1969)
- Ya.B. Zeldovich, A.A. Rusmaikin, D.D. Sokoloff, Magnetic fields in astrophysics. The Fluid Mechanics of Astrophysics and Geophysics (Gordon and Breach, New York, 1983)
- G.M. Vodinchar, L.K. Feshchenko, Bulletin KRASEC. Phys. and Math. Sci, Mathematical modeling, **2(11)**, 42-54 (2015)
- G.M. Vodinchar, Bulletin KRASEC. Phys. and Math. Sci, Mathematical modeling, **2(7)**, 33-42 (2013)
- E.N. Parker, Astrophys. J., **122**, 293-314 (1955)
- A.N. Godomskaya, O.V. Sheremetyeva, E3S Web of Conferences, **62**, 02016 (2018)
- G.M. Vodinchar, A.N. Godomskaya, O.V. Sheremetyeva, Bulletin KRASEC. Phys. and Math. Sci, Mathematical modeling, **2(7)**, 33-42 (2018)
- A.N. Godomskaya, O.V. Sheremetyeva, E3S Web of Conferences, **127**, 02016 (2019)
- A.N. Godomskaya, O.V. Sheremetyeva, Bulletin KRASEC. Phys. and Math. Sci, Mathematical modeling, **4(29)**, 58-66 (2019)

**Thanks for your attention**